AAE 637 Lab 6: Post Estimation Topics

TA: Charng-Jiun Yu

February 28, 2018

1 Multiple Hypothesis Test with Nonlinear Models

Multiple hypothesis test, or joint hypothesis test, is done by testing several restrictions at the same time using a single test statistic. A natural question is: why can't we just test each restriction separately? The answer is yes, we can certainly do that. However, there are some reasons we might prefer doing a joint test:

1. Highly correlated variables:

   Suppose that we are estimating a linear regression model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$, where x_1 and x_2 are highly correlated. When we test whether each of them has an impact on y, we might be unable to reject the null hypothesis for both x_1 and x_2 if testing them separately due to high standard errors, but are able to reject the null hypothesis if testing them jointly. This is because separate t-tests ignore the correlated structure between two variables.

2. Controlling type I error:

   For most of the hypothesis testing, we are controlling the type I error. As you all know, a significance level of 0.05 indicates a 5% chance that type I error occurs. If we do 10 separate t-tests, the probability that at least a type I error occurs is $1-(1-\alpha)^{10} \approx 40\%$. This can be quite high. If we are doing a joint hypothesis test, the probability is still 5%. Therefore, if we are really concerned about the type I error but do not want to lower the significance level in each individual test, we might want to do a single joint test.

- The model we are estimating here is the following:

  $$q_b = \beta_1 p_b^{\beta_2} p_l^{\beta_3} p_o^{\beta_4} I^{\beta_5} + u$$

  where q_b is the quantity demand of beer, p_b is the price of beer, p_l is the price of liquor, p_o is the price of other goods, and I is the income. Note that the log linearizing trick does not work here because the error term is additive. We have to estimate this model nonlinearly.

  Our goal is to jointly test the following three restrictions:

  $$H_0: \begin{cases} \beta_2 + \beta_3 + \beta_4 + \beta_5 = 0 \\ \beta_2 = -\beta_3 \\ \beta_5 = 1 \end{cases}$$

The first restriction says that all elasticities sum up to zero. The second restriction is the condition that the own-price elasticity of beer is equal to the negative of the cross-price elasticity of beer with respect to the price of liquor. The last restriction indicates that the income elasticity is one. This joint test can be done by calculating the F-stat that can be used to compare the SSEs from the unrestricted model (full model) and the restricted model (when the restrictions hold). The idea is that if the null hypothesis is true, then the SSEs of unrestricted model and the restricted model should be very closed. How closed they are is determined by the F-stat. Note that the SSE for unrestricted model must be smaller or equal to that for the restricted model because unrestricted model gives us a larger set of possible parameter values, including the values when the restrictions hold, that can give us smaller SSE.

We can get the restricted model by simply plugging the restrictions in the original model:

$$q_b = \beta_1 p_b^{\beta_2} p_l^{-\beta_2} p_o^{-1} I^1 + u$$

After estimating two models, we can get the SSE required to calculate the F-stat:

$$F_{J,N-K} = \frac{\frac{SSE_r - SSE_{ur}}{J}}{\frac{SSE_{ur}}{N-K}}$$

where J is the number of restrictions, N is the number of observations, and K is the number of parameters (including intercept) for the unrestricted model. In our application, $J = 3$, $K = 5$, and $N = 30$.

Our F-stat is 127.8195, which is greater than the critical value under 0.05 significance level, 2.9912. Therefore, we reject the joint hypothesis that $\beta_2 + \beta_3 + \beta_4 + \beta_5 = 0$, $\beta_2 = -\beta_3$, and $\beta_5 = 1$ are all true.

2 Statistics Evaluated at Different Evaluation Points

Many statistics depend on points of evaluation. A common example is the own-price elasticity $\epsilon = \frac{\partial q}{\partial p} \cdot \frac{p}{q}$, where the value of ϵ can be different across q and p. The following example will show you how much different it can make when we evaluate a statistics at different evaluation points.

- Consider the following CES production function:

$$y = \alpha[\delta L^{-\rho} + (1-\delta)K^{-\rho}]^{-\frac{\eta}{\rho}} \exp(u)$$

where y is output quantity, L is labor, and K is capital. Each parameter has there own economic interpretation: α is the production efficiency, δ is the distribution parameter, η is the returns to scale parameter, and ρ is the substitution parameter. As the name constant elasticity of substitution suggests, the intuition of CES production function is that the the elasticity of the ratio of two inputs with respect to the ratio of their marginal products is constant ($\frac{1}{1+\rho}$). You might also see people use CES utility functions.

It is easier to estimate the function by taking the log:

$$\log(y) = \log(\alpha) - \frac{\eta}{\rho} \log[\delta L^{-\rho} + (1-\delta)K^{-\rho}] + u$$

Note that this is still a nonlinear function with respect to parameters.

- The statistics of interest here is the marginal rate of substitution, MRS, which indicates the ratio of marginal production between labor and capital:

$$MRS_{LK} = \frac{\frac{\partial y}{\partial K}}{\frac{\partial y}{\partial L}} = (\frac{1-\delta}{\delta})(\frac{L}{K})^{1+\rho}$$

  We can see that MRS depends on K and L in this case.

- After completing the estimation, let's see the MRS under different L and K. We evaluate in four ways:
  (1) At each observation and then take the average
  (2) At the mean of L and K
  (3) At the minimum value of L and the associated K for that observation
  (4) At the maximum value of L and the associated K for that observation

Table 2.1: MRS at Different Evaluation Points

	MRS	Standard Error	t-stat against 1	Test on MRS=1
Average across all observations	55196261	726234466	0.0760	Do not reject
Mean of data	2.7195	2.2197	0.7746	Do not reject
Min labor and its associated capital	0.0000000019	0.0000000236	-42389283	Reject
Max labor and its associated capital	18384016	1918105552	0.0958	Do not reject

We can see that the values of MRS are very different across evaluation points. This also affects the result of hypothesis test on MRS=1 under 0.05 significance level. Moral of the story: where we evaluate a statistic matters.

⋆ Data for two examples are both from Griffiths, Hill, and Judge (1993)'s econometrics textbook. This is like an undergrad version of Judge et al.